MATH 590: QUIZ 9 SOLUTIONS

Name:

1. Let V be a finite dimensional vector space with inner product $\langle\ ,\ \rangle$ and orthonormal basis $u_1, \ldots, u_n$.

   (i) $\langle v_1 + v_2, w\rangle = \langle v_1, w\rangle + \langle v_2, w\rangle$. (2 points)

   (ii) For $v \in V$, use the inner product to write v as a linear combination of $u_1, \ldots, u_n$. (2 points)
   Solution for (ii). $v = \langle v, u_1\rangle \cdot u_1 + \cdots + \langle v, u_n\rangle \cdot u_n$.

2. Let V denote the vector space of real polynomials having degree less than or equal to one, with inner product $\langle f(x), g(x)\rangle := \int_{-1}^{1} f(x)g(x)\ dx$. Show that $f_1 := \frac{1}{\sqrt{2}}$ and $f_2 := \sqrt{\frac{3}{2}}x$ is an orthonormal basis for V. (6 points).

Solution. $\int_{-1}^{1} f_1(x) \cdot f_2(x) = \int_{-1}^{1} \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{3}{2}}x\ dx = \frac{\sqrt{3}}{2}(\frac{x^2}{2})_{-1}^{1} = 0$, so $f_1(x)$ and $f_2(x)$ are orthogonal.
$||f_1|| = \{\langle f_1, f_1\rangle\}^{1/2} = \{\int_{-1}^{1} \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\ dx\}^{1/2} = \{(\frac{x}{2})_{-1}^{1}\}^{1/2} = 1$, so that f_1 has length one.
$||f_2|| = \{\langle f_2, f_2\rangle\}^{1/2} = \{\int_{-1}^{1} \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{\sqrt{2}}x\ dx\}^{1/2} = \frac{\sqrt{3}}{2}(\frac{x^2}{2}{}_{-1}^{1} = 0$, so f_1, f_2 are orthogonal.
Therefore, f_1, f_2 is an orthonormal system. Since such a system is linearly independent, and V has dimension two, it follows that f_1, f_2 is an orthonormal basis for V.